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Dynamically stable control of articulated crowds

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ABSTRACT

The synthesis of realistic complex body movements in real-time is a difficult problem in computer graphics and in robotics. High realism requires the accurate modeling of the details of the trajectories for a large number of degrees of freedom. At the same time, real-time animation necessitates flexible systems that can adapt and react in an online fashion to changing external constraints. Such behaviors can be modeled with nonlinear dynamical systems in combination with appropriate learning methods. The resulting mathematical models have manageable mathematical complexity, allowing to study and design the dynamics of multi-agent systems. We introduce Contraction Theory as a tool to treat the stability properties of such highly nonlinear systems. For a number of scenarios we derive conditions that guarantee the global stability and minimal convergence rates for the formation of coordinated behaviors of crowds. In this way we suggest a new approach for the analysis and design of stable collective behaviors in crowd simulation.

1. Introduction

The generation of realistic interactive human movements is a difficult task with high relevance for computer graphics and robotics. Applications such as computer games require online synthesis of such movements, at the same time providing high degrees of realism even for complex body movements. While for the off-line synthesis of human movements, the movements can be recorded off-line and retargeted to the relevant kinematic model, this procedure is not possible for online synthesis. Approaches based on physical or dynamical models (e.g. [1]) have focused on the simulation of scenes with many interacting agents that navigate autonomously and show interesting collective behaviors. Due to the complexity of the underlying mathematical models, such systems are typically designed in a heuristic manner. Opposed to other applications in engineering, the system dynamics of such computer animation systems is usually not analyzed, so that robustness or stability guarantees for the system dynamics cannot be given.

In this paper we present first steps toward the development of more systematic method for the design of the dynamics of interactive crowds. For this purpose, we approximate human movements by relatively simple mathematical models, combining dynamical models with appropriate learning methods. In addition, we introduce Contraction Theory [2] as a new tool for the stability design of complex nonlinear dynamical systems. We demonstrate how this approach can be applied for the stability analysis of groups of autonomously navigating characters with full body articulation. The current research extends our previous work [3] by the development of stability analysis for more complex scenarios, including the control of heading direction and of the consensus behavior of crowds.

The paper is structured as follows: The learning-based dynamical model for complex human movements is briefly sketched in Section 3. Section 4 describes the relevant control dynamics that is required to realize complex interactions between different characters in a crowd. In Section 5 we review basic concepts from Contraction Theory. The major results of our stability analysis and some demonstrations of their application to crowd animation are presented in Section 6, followed by some conclusions.

2. Related work

Dynamical systems have been frequently applied in crowd animation for the simulation of autonomous collective behaviors (e.g. [4,5]). Some of this work was inspired by observations in biology showing that coordinated behavior of large groups of agents, such as flocks of birds, can be modeled as emergent behaviors arising

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by self-organization from dynamical couplings between interacting agents [6–8].

Simple rules for such dynamic interactions have been derived from the observation of the motion of flocks of birds [9], such as: collision avoidance by nearest neighbor distance control, velocity matching and flock centering. The obtained models for the self-organization of behavior have been analyzed by application of methods from nonlinear dynamics [10]. They are interesting for computer animation because they permit to reduce the computational costs of traditional computer animation techniques, such as scripting or path planning [5]. In addition, the generation of collective behavior by self-organization has the advantage that the system can adapt autonomously to external perturbations or changes in the system architecture, such as the variation of the number of characters [7]. However, the mathematical analysis of the underlying dynamical systems is typically quite complicated. The dynamics describing individual agents is typically highly nonlinear [11], making a systematic treatment of stability properties often infeasible, even for individual characters. In addition, crowd animation requires the dynamic interaction of many such agents. The control of collective behaviors of groups of agents has been treated in mathematical control theory [12,13], however typically assuming highly simplified often even linear models for the agents. Group coordination and cooperative control have also been studied in robotics, e.g. in the context of the navigation of groups of vehicles [14,15], or with the goal to generate collective behavior by self-organization. Examples are the spontaneous adaptation to perturbations of inter-agent communication or changes in the number of agents [16,17]. Many approaches have only analyzed asymptotic stability for consensus scenarios, while exponential stability what also allows to derive bounds for the rate of convergence has rarely been treated [15]. The same result can be derived by application of Contraction Theory [18], a method for the derivation of conditions for the uniform exponential convergence of complex nonlinear systems. Opposed to classical stability analysis for nonlinear systems, Contraction Theory permits to re-use stability results for system components in order to derive conditions that guarantee the stability of the overall system. It provides a useful tool specifically for modularity-based stability analysis and design. Contraction Theory has been already successfully implemented for synchronization of DMPs (dynamic movement primitives) controlling Unmanned Aerial Vehicles [19]. But the stability properties for crowd animation systems realizing human behaviors with realistic levels of complexity have basically never been treated before.

3. System architecture

The proposed new method for the design of the collective dynamics of interacting crowds is based on a learning-based approach for the modeling of human movements using dynamic movement primitives [20] (cf. Fig. 1). For a relevant class of movements, like gaits with different styles or straight vs. curved locomotion, a low-dimensional representation in terms of a small number of basic components is learned using an algorithm for anechoic demixing [21]. We showed elsewhere that this method allows to approximate trajectories by very small number of source signals, outperforming other dimension reduction algorithms, such as ICA or PCA [22,21].

In order to generate the learned source signals online, the stable solutions of a nonlinear dynamical system (*dynamic primitive*) are mapped onto the form of the source functions. For the synthesis of gait pattern a primitive is naturally modeled by oscillator [23]. The mapping from the phase space of the dynamics, defined by the state variables $\mathbf{y} = [y, \dot{y}]^T$, onto the values of the source functions s_j was learned from training data using Support Vector Regression with gaussian kernel (see Ref. [20] for details). For the examples presented in this paper each character was modeled by a single Andronov–Hopf oscillator. The generated source signals were then linearly combined with the linear weights w_{ij} and phase delays τ_{ij} in order to generate the joint angle trajectories $\xi_i(t)$ according to the learned *anechoic mixture model* that was given by the equation:

$$\xi_i(t) = \sum_j w_{ij} s_j \left(t - \tau_{ij} \right) \quad (1)$$

The complete reconstruction of the trajectories requires the addition of the average joint angles m_i, which also were learned from the training data.

For the special case where the dynamic primitives are given by Hopf oscillators, whose limit cycle for appropriate choice of the coordinate system is a circular trajectory in phase space, the phase delays can be absorbed in an instantaneous orthogonal mapping (rotation) $\mathbf{M}_{\tau_{ij}}$ in the phase plane of the oscillator (Fig. 1b). This allows to derive dynamics for online synthesis without explicit delays, which would greatly complicate the system dynamics. (See Ref. [20] for further details.)

By blending of the mixing weights w_{ij} and the phase delays τ_{ij}, intermediate gait styles can be generated. This technique was applied to generate walking along paths with different curvatures, changes in step length, and emotional gait styles. Interactive behavior of multiple characters can be modeled by making the states of the oscillators and the mixing weights dependent on the behavior of the other characters. Such couplings, which are discussed in more detail in the next section, result in a highly nonlinear overall system dynamics. We showed elsewhere that the same architecture can be applied also for the generation of other types of body motion than locomotion [20], while such examples are not discussed in this paper. The walking direction of the characters is also changed by interpolation between straight walking and walking along curved paths to the left or to the right. The parameters used for blending are described in our previous publications [20,24]. Such blending is used to simulate a control of the heading directions in consensus scenarios and for obstacle avoidance during the autonomous reordering of crowds. For the implementation of reactive local obstacle avoidance we used a dynamic navigation model that originally was developed in robotics [25]. See Refs. [24,20] for details concerning the implementation.

4. Control dynamics

Flexible control of the locomotion of articulating agents requires the control of multiple variables, specifying a control dynamics with multiple coupled levels. For the examples discussed in this paper our system included the control of the following variables: (1) phase within the step cycle, (2) step length, (3) gait frequency, and (4) heading direction. The control of step phase was accomplished by coupling of the Andronov–Hopf oscillators [26] that correspond to different agents, resulting in phase synchronization. These oscillators have a stable limit cycle that corresponds to an oscillation with constant amplitude and the (time-dependent) phase $\phi(t)$. In absence of external couplings the phase increases linearly, i.e. $\phi(t) = \omega t + \phi(0)$, where ω is the stable eigenfrequency of the oscillator. Control of step frequency was accomplished by varying this parameter in a time-dependent manner in dependence of the behavior of the characters in the scene. Step-length and direction were controlled by morphing between gaits with different step lengths or path curvatures, blending the parameters of the anechoic mixing model (see above). In this case the controlled variables are the blending coefficient of these mixtures. (See Ref. [20] for details.)

The formulation of the system dynamics in terms of speed control is simplified by the introduction of the positions z_i for

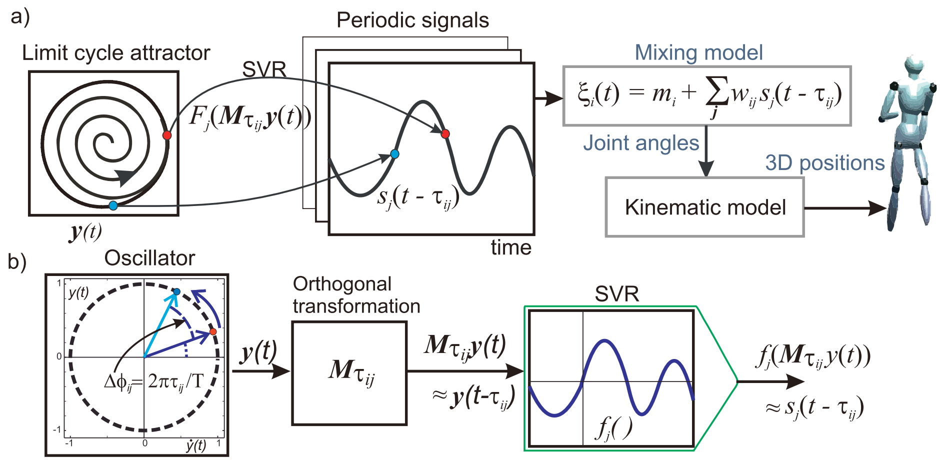

Fig. 1. (a) Architecture of the system for real-time synthesis of complex human movements. Solutions of dynamical systems (primitives) are mapped onto source signals, that have been derived by anechoic demixing from training data. The solutions of the dynamical systems are mapped by Support Vector Regressions (SVR) onto the source signals. These source signals are then combined using the learned anechoic mixing model to generate joint angle trajectories online, which specify the kinematics of the animated characters. (b) When the dynamic primitives are modeled by nonlinear oscillators the time shifts of the anechoic mixing model can be absorbed in an instantaneous orthogonal transformation $\mathbf{M}_{\tau_{ij}}$, avoiding a dynamics with explicit delays.

each individual character along its propagation path (see Fig. 2). This variable fulfills the differential equation $\dot{z}_i(t) = \dot{\phi}_i g(\phi_i)$, where the positive function g determines the instantaneous propagation speed of the character depending on the phase within the gait cycle. This nonlinear function was determined empirically from a kinematic model of a character. By integration of this propagation dynamics one obtains $z_i(t) = G(\phi_i(t) + \phi_i^0) + c_i$, with an initial phase shift ϕ_i^0 and some constant c_i depending on the initial position and phase of avatar i, and the monotonously increasing function $G(\phi_i) = \int_0^{\phi_i} g(\phi)d\phi$, where we assume $G(0) = 0$.

In the following we will analyze four different control rules, whose combination allows to generate quite flexible locomotion behavior of a crowd of characters:

(I) Control of step frequency: a simple form of speed control results if the frequency of the oscillators $\dot{\phi}_i$ is made dependent on the behavior of the other characters. Assuming that ω_0 be the equilibrium frequency of the oscillators without interaction, this can be accomplished by the control dynamics:

$$\dot{\phi}_i(t) = \omega_0 - m_d \sum_{j=1}^{N} K_{ij}[z_i(t) - z_j(t) - d_{ij}] \tag{2}$$

The constants d_{ij} specify the stable pairwise relative distances in the final formed order for each pair (i, j) of characters. The elements of the coupling graph's adjacency matrix $\mathbf{K}$ determine whether characters i and j are interacting and thus dynamically coupled. These parameters were set to $K_{ij} = 1$, if the characters were coupled, and they are zero otherwise (with $K_{ii} = 0$). For example, we choose $K_{ij} = 1$, $\forall\, i \neq j$ for *all-to-all coupling*, and $K_{ij} = 1$, $\forall$ mod$(|i - j|, N) = 1$ for *ring coupling*. The constant $m_d > 0$ determines the coupling strength.

With the Laplacian matrix $\mathbf{L}^d$ of the coupling graph (that is assumed to be *strongly connected* [14,27,28]), defined by $L_{ij}^d = -K_{ij}$ for $i \neq j$ and $L_{ii}^d = \sum_{j=1}^N K_{ij}$, and the constants $c_i = -\sum_{j=1}^N K_{ij} d_{ij}$, the last equation system can be re-written in vector form:

$$\dot{\phi} = \omega_0 \mathbf{1} - m_d(\mathbf{L}^d G(\phi + \phi^0) + \mathbf{c}) \tag{3}$$

(II) Control of step length: step length was varied by morphing between gaits with short and long steps. A detailed analysis showed that the influence of step length on propagation speed could be well approximated by simple linear rescaling. If the propagation velocity of character i is $v_i(t) = \dot{z}_i(t) = \dot{\phi}_i(t)g(\phi_i(t)) = \omega_i(t)g(\phi_i(t))$ for the normal step size, then the velocity for modified step size could be approximated by $v_i(t) = \dot{z}_i(t) = (1 + \mu_i)\omega_i(t)g(\phi_i(t))$ with the morphing parameter μ_i. The empirically measured propagation velocity as function of gait phase is shown in Fig. 3(a) for different values of the step length parameter μ_i.

In order to realize speed control by step length the morphing parameter μ_i was made dependent on the difference between

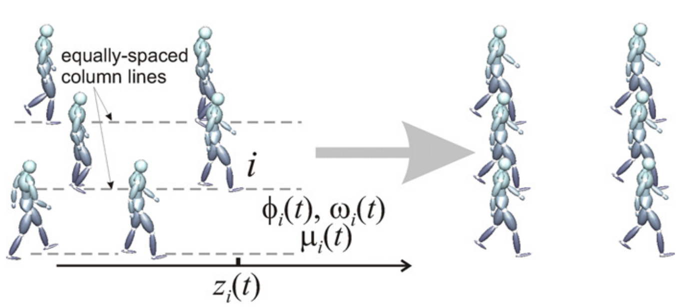

Fig. 2. Variables exploited for speed and position control. Every character i is characterized by its position $z_i(t)$, the phase $\phi_i(t)$ and the instantaneous eigenfrequency $\omega_i(t) = \dot{\phi}_i(t)$ of the corresponding Andronov–Hopf oscillator, and a step-size scaling parameter $\mu_i(t)$.

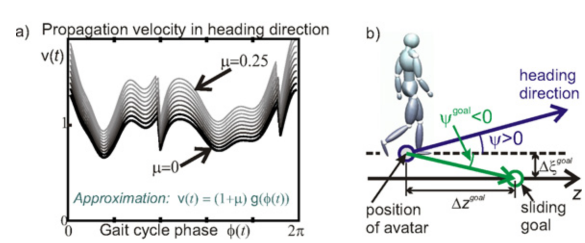

Fig. 3. (a) Propagation velocity for different values of the step-length morphing parameter ($\mu = [0\ldots0.25]$) as function of the gait cycle phase ϕ. Empirical estimates are well approximated by a linear rescaling of the propagation speed function defined above $\dot{z} \approx (1 + \mu)g(\phi)$, for constant $\omega = 1$. (b) The heading direction control depends on the difference between the actual heading direction ψ^{heading} and the goal direction ψ^{goal}. Movement along parallel lines was modeled by defining 'sliding goals' that moved along the lines.

actual and desired position differences d_{ij} between the agents $\mu_i = -m_z \sum_{j=1}^{N} K_{ij}^z [z_i(t) - z_j(t) - d_{ij}]$, resulting in the control rule:

$$\dot{z}_i(t) = \omega_i(t) g(\phi_i(t))(1 - m_z \sum_{j=1}^{N} K_{ij}^z [z_i(t) - z_j(t) - d_{ij}])$$

with the constant coupling strength $m_z > 0$. Here the adjacency matrix $\mathbf{K}^z$ of the coupling graph corresponds to the Laplacian matrix $\mathbf{L}^z$ (according to the equivalent relationships as specified above). In vector notation the dynamics for the control of speed by step length can be written:

$$\dot{\mathbf{z}} = \omega g(\phi + \phi^0)(1 - m_z(\mathbf{L}^z \mathbf{z} + \mathbf{c})) \tag{4}$$

(III) Control of step phase: by defining separate controls for step length and step frequency the position and step phase of the characters can be varied independently. This makes it possible to simulate arbitrary spatial patterns of characters, at the same time synchronizing their step phases. The additional control of step phase can be accomplished by simple addition of a linear coupling term in Eq. (3):

$$\dot{\phi} = \omega_0 \mathbf{1} - m_d(\mathbf{L}^d G(\phi + \phi^0) + \mathbf{c}) - k \mathbf{L}^\phi \phi \tag{5}$$

with $k > 0$ and the Laplacian $\mathbf{L}^\phi$. (All sums or differences of angular variables were computed by modulo 2π.)

(IV) Control of heading direction: the control of the heading directions ψ_i of the characters was based on differential equations that specify attractors for goal directions ψ_i^{goal}, which were computed from 'sliding goals' that were placed along straight lines at fixed distances in front of the characters (Fig. 3b). The heading dynamics was given by a nonlinear differential equation, independently for every character [20]:

$$\dot{\psi}_i = \omega_i(t)(-m_\psi \sin(\psi_i - \psi_i^{\text{goal}}) + g^\psi(\phi_i(t) + \phi_i^0)) \tag{6}$$

where $\psi_i^{\text{goal}} = \arctan(\Delta\xi_i^{\text{goal}} / \Delta z_i^{\text{goal}})$, with $\Delta\xi_i^{\text{goal}}$ specifying the distance to the goal line orthogonal to the propagation direction and Δz_i^{goal} being a constant (Fig. 3b). The first term describes a simple direction controller whose gain is defined by the constant $m_\psi > 0$. The second term approximates oscillations of heading direction, where g^ψ is again an empirically determined periodic function. Control is realized by making the morphing coefficients that determine the contributions of left vs. right-curved walking dependent on the change rate $\dot{\psi}_i$ of the heading direction.

The mathematical results derived in the following sections apply to subsystems derived from the complete system dynamics defined by Eqs. (4), (5) and (6). In addition, simulations will be presented that illustrate the range of behaviors that can be modeled by the full system dynamics.

5. Elements from Contraction Theory

The dynamical systems for the modeling of the behavior of the autonomous characters derived in the last section are essentially nonlinear. In contrast to linear dynamical systems, a major difficulty of the analysis of such nonlinear systems is that stability properties of systems parts usually do not transfer to composite systems. *Contractiontheory* (CT) [2] provides a general method for the analysis of essentially nonlinear systems that permits such a transfer. This makes it suitable for the analysis of complex systems that are composed from components. CT characterizes the system stability by the behavior of the differences between solutions with different initial conditions. If these differences vanish exponentially over time independent from the chosen initial states the system is called *contracting*. In this case the system is *globally asymptotically stable*, that is all its solutions converge to a single trajectory independent from the initial state. For a general dynamical system of the form

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) \tag{7}$$

we assume that $\mathbf{x}(t)$ is one solution of the system and $\tilde{\mathbf{x}}(t) = \mathbf{x}(t) + \delta\mathbf{x}(t)$ a neighboring one with different initial condition. The function $\delta\mathbf{x}(t)$ is also called *virtual displacement*. With the Jacobian of the system $\mathbf{J}(\mathbf{x}, t) = \partial\mathbf{f}(\mathbf{x}, t)/\partial\mathbf{x}$ it can be shown [2] that any virtual displacement decays exponentially to zero over time if the symmetric part of the Jacobian $\mathbf{J}_s = (\mathbf{J} + \mathbf{J}^T)/2$ is uniformly negative definite, denoted as $\mathbf{J}_s < 0$, i.e. has negative eigenvalues for all relevant state vectors $\mathbf{x}$. In this case, it can be shown that the norm of the virtual displacement decays at least exponentially to zero for $t \to \infty$. If the virtual displacement is small enough, then

$$\frac{d}{dt}\delta\mathbf{x}(t) = \mathbf{J}(\mathbf{x}, t)\delta\mathbf{x}(t)$$

implies through $\frac{d}{dt}||\delta\mathbf{x}(t)||^2 = 2\delta\mathbf{x}^T(t)\mathbf{J}_s(\mathbf{x}, t)\delta\mathbf{x}$ the inequality: $||\delta\mathbf{x}(t)|| \le ||\delta\mathbf{x}(0)|| e^{\int_0^t \lambda_{\max}(\mathbf{J}_s(\mathbf{x}, s))ds}$. The decay of the virtual displacement occurs thus with a *convergence rate* (inverse timescale) that is bounded from below by the *contraction rate* $\rho_c = -\sup_{\mathbf{x},t} \lambda_{\max}(\mathbf{J}_s(\mathbf{x}, t))$, where $\lambda_{\max}(.)$ signifies the largest eigenvalue. This has the consequence that all trajectories converge to a single solution exponentially in time [2].

Contraction analysis can be applied to *hierarchically coupled systems* that are given by the dynamics

$$\frac{d}{dt}\begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{f}_1(\mathbf{x}_1) \\ \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) \end{pmatrix} \tag{8}$$

where the first subsystem is not influenced by the state of the second. The corresponding Jacobian $\mathbf{F} = \begin{pmatrix} \mathbf{F}_{11} & 0 \\ \mathbf{F}_{21} & \mathbf{F}_{22} \end{pmatrix}$ implies for the dynamics of the virtual displacements: $\frac{d}{dt}\begin{pmatrix} \delta\mathbf{x}_1 \\ \delta\mathbf{x}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{F}_{11} & 0 \\ \mathbf{F}_{21} & \mathbf{F}_{22} \end{pmatrix}\begin{pmatrix} \delta\mathbf{x}_1 \\ \delta\mathbf{x}_2 \end{pmatrix}$. If $\mathbf{F}_{21}$ is bounded, then the exponential convergence of the first subsystem, (following from $(\mathbf{F}_{11})_s < 0$), implies thus convergence of the whole system, if in addition $(\mathbf{F}_{22})_s < 0$. This follows from the fact the term $\mathbf{F}_{21}\delta\mathbf{x}_1$ is just an exponentially decaying disturbance for the second subsystem. (See Ref. [2] for details of proof.)

In practical applications many systems are not contracting with respect to all dimensions of the state space, but rather show convergence only with respect to a subset of dimensions. This behavior can be mathematically characterized by *partial contraction* [18,28]. The underlying idea is the construction of an auxiliary system that is contracting with respect to a subset of dimensions (or submanifold) in state space. The major result is the following [18]:

Theorem 1. *(Partial contraction) Consider a nonlinear system of the form* $\dot{\mathbf{x}} = \mathbf{f}(\mathrm{x}, \mathrm{x}, t)$ *and the auxiliary system* $\dot{\mathbf{y}} = \mathbf{f}(\mathrm{y}, \mathrm{x}, t)$. *If the auxiliary system is contracting with respect to* $\mathbf{y}$ *uniformly for all relevant* $\mathbf{x}$ *then the original system is called partially contracting. This implies that if a particular solution of the auxiliary system verifies a specific smooth property then all trajectories of the original system also verify this property with exponential convergence.*

A 'smooth property' is a property of the solution that depends smoothly on space and time (assuming the relevant derivatives or partial derivatives exist and are continuous), such as convergence against a particular solution or a properly defined distance to submanifold in phase space ([18]).

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It thus is sufficient to show that the auxiliary system is contracting to prove convergence to a subspace. If the original system has a flow-invariant *linear subspace* $\mathcal{M}$, which is defined by the property that trajectories starting in this space always remain in it ($\forall t$: $\mathbf{f}(\mathcal{M}, t) \subset \mathcal{M}$), and assuming that the matrix $\mathbf{V}$ is an orthonormal projection onto $\mathcal{M}^{\perp}$, then a sufficient condition for global exponential convergence to $\mathcal{M}$ is given by [27,28]:

$$\mathbf{V}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right)_s \mathbf{V}^T < \mathbf{0}, \qquad (9)$$

where $\mathbf{A} < \mathbf{0}$ again indicates that the matrix $\mathbf{A}$ is negative definite.

Finally, we introduce here a theorem that provides sufficient conditions for synchronization of a network that is composed from N identical dynamical systems that communicate through a common medium or channel with state variable χ. The relevant dynamics is given by

$$\begin{array}{l} \dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \chi, t), \\ \dot{\chi} = g(\chi, \Psi(\mathbf{x}), t) \end{array}, \qquad (10)$$

$\mathbf{x}$ containing the state variables of the individual systems and all components of $\mathbf{f}$ having the same form f. Exploiting the last Partial contraction theorem the following result can be derived [29]:

Theorem 2. *(Quorum sensing) If the reduced order virtual system $\dot{y} = f(y, \chi, t)$ is contracting for all relevant χ then all solutions of the original system converge exponentially against a single trajectory, i.e. $|x_i(t) - x_j(t)| \to 0$ as $t \to +\infty$.*

6. Results: stability conditions for crowd control

In the following we derive stability conditions for the formation of coordinated behavior of crowds, providing contraction bounds for four scenarios corresponding to control problems with increasing levels of complexity. Corresponding crowd behaviors are illustrated by demo movies that are provided as supplements for the manuscript.

(1) Control of step phase without position control: this simple control rule permits to simulate step synchronization, as in the case of a group of soldiers [28], [**Demo**[1]]. The dynamics for this case is given by Eq. (5) with $m_d = 0$ (omitting the position control term). For N identical dynamical systems with symmetric identical coupling gains $K_{ij} = K_{ji} = k$ the dynamics can be written

$$\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i) + k \sum_{j \in \mathcal{N}_i} \left(\mathbf{x}_j - \mathbf{x}_i\right), \quad \forall i = 1, \ldots, N \qquad (11)$$

where $\mathcal{N}_i$ defines the index set specifying the neighborhood in the coupling graph, i.e. the other characters that are directly interacting with character i. The system can be rewritten compactly: $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, t) - k\mathbf{L}\mathbf{x}$ with the concatenated phase variable $\mathbf{x} = [\mathbf{x}_1^T, \ldots, \mathbf{x}_N^T]^T$. The matrix $\mathbf{L} = \mathbf{L}_G \bigotimes I_p$ is derived from the *Laplacian matrix* of the coupling graph $\mathbf{L}_G$, where p is the dimensionality of the individual sub-systems (I_p is the identity matrix of dimension p, and $\bigotimes$ signifies the *Kronecker product*). The Jacobian of this system is given by $\mathbf{J}(\mathbf{x},t) = \mathbf{D}(\mathbf{x},t) - k\mathbf{L}$, where the block-diagonal matrix $\mathbf{D}(\mathbf{x}, t)$ contains the Jacobians of the uncoupled components $\frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x}_i, t)$.

The dynamics has a flow-invariant linear subspace $\mathcal{M}$ that contains the particular solution $\mathbf{x}_1^* = \cdots = \mathbf{x}_n^*$. For this solution all state variables $\mathbf{x}_i$ are identical and thus in synchrony. In this case, the coupling term in Eq. (11) vanishes, so that the form of the solution is identical to the one of an uncoupled system $\dot{\mathbf{x}}_i = \mathbf{f}(\mathbf{x}_i)$. If $\mathbf{V}$ is a projection matrix onto the invariant subspace $\mathcal{M}^{\perp}$, then by Eq. (9) the sufficient condition for convergence toward $\mathcal{M}$ is given by $\mathbf{V}(\mathbf{D}(\mathbf{x}, t) - k\mathbf{L})_s\mathbf{V}^T < \mathbf{0}$ [28]. This implies $\lambda_{\min}\left(\mathbf{V}(k\mathbf{L})_s\mathbf{V}^T\right) = k\lambda_{\mathbf{L}}^+ > \sup_{\mathbf{x},t}\lambda_{\max}(\mathbf{D}_s)$, with $\lambda_{\mathbf{L}}^+$ being the smallest non-zero eigenvalue of symmetrical part of the Laplacian $\mathbf{L}_s$. (For strongly connected coupling graphs all the nonzero eigenvalues of $\mathbf{L}_s$ are real positive, due to the Gershgorin's disc theorem [30].) The sufficient condition for global stability of the overall system is given by $k > \sup_{\mathbf{x},t}\lambda_{\max}\left(\frac{\partial \mathbf{f}}{\partial \mathbf{x}}(\mathbf{x}, t)\right)/\lambda_{\mathbf{L}}^+$. This implies the minimum convergence rate: $\rho_c = -\sup_{\mathbf{x},t}\lambda_{\max}(\mathbf{V}(\mathbf{D}(\mathbf{x}, t) - \mathbf{L})_s\mathbf{V}^T)$. Different topologies of the coupling graphs result in different Laplacians and thus stability conditions. For example $\lambda_{\mathbf{L}}^+ = 2(1 - \cos(2\pi/N))$ for symmetric coupling of the ring topology, and $\lambda_{\mathbf{L}}^+ = N$ for all-to-all coupling, where N is the number of avatars. (See Refs. [18] and [28] for more details.) In the particular case of Eq. (11) with $m_d = 0$, for the phase coupling of Hopf oscillators with $\mathbf{x}_i = \phi_i$, we have $\mathbf{f}(\mathbf{x}_i) = \omega_0 = \text{const}$ and the contraction condition becomes $k\lambda_{\mathbf{L}}^+ > 0$ with the uniform contraction rate $\rho_c = k\lambda_{\mathbf{L}}^+$ for $k > 0$.

(2) Speed control by variation of step frequency: the dynamics of this scenario is given by Eqs. (3) and (4) for $m_z = 0$. Assuming arbitrary initial distances and phase offsets of different propagating characters, implying by $G(\phi_i^0) = c_i$ that $c_i \neq c_j$, for $i \neq j$, we redefine d_{ij} as $d_{ij} - (c_i - c_j)$ in Eq. (2), and accordingly $\mathbf{c}$ in Eq. (3). Furthermore, we assume for this analysis a scenario where the characters follow one leading character whose dynamics does not receive input from the others. In this case all phase trajectories converge to a single unique trajectory only if $c_i = c_j$ for all i, j, as consequence of the strict correspondence between gait phase and position that is given by Eq. (3). In all other cases the trajectories of the followers converge to one-dimensional, but distinct attractors that are uniquely defined by c_i. These attractors correspond to a behavior where the followers' positions oscillate around the position of the leader. The partial contraction of the dynamics with $\mathbf{c} = 0$ guarantees that the resulting attractor area is bounded in phase space (cf. Ch. *3.7.vii* in Ref. [2]).

For the analysis of contraction properties we regard an auxiliary system obtained from Eq. (3) by keeping only the terms that depend on ϕ: $\dot{\phi} = -m_d\mathbf{L}^d G(\phi + \phi^0)$. According to Theorem 1 the symmetrized Jacobian of this system projected onto the orthogonal complement of the flow-invariant linear subspace $\phi_1^* + \phi_1^0 = \ldots = \phi_N^* + \phi_N^0$ determines whether this system is partially contracting. By virtue of a linear change of variables, the study of the contraction properties of this system is equivalent to study the contraction properties of the dynamical system $\dot{\phi} = -m_d\mathbf{L}^d G(\phi)$ on trajectories converging toward the flow-invariant manifold $\phi_1^* = \ldots = \phi_N^*$.

The sufficient conditions for (exponential) partial contraction toward flow-invariant subspace are, (see Eq. (9)): $\mathbf{V}\mathbf{J}_s(\phi)\mathbf{V}^T = -m_d\mathbf{V}\mathbf{B}(\phi)\mathbf{V}^T < 0$, introducing $\mathbf{B}(\phi) = \mathbf{L}^d\mathbf{D}_g + \mathbf{D}_g(\mathbf{L}^d)^T$ and $\mathbf{V}$ signifying the projection matrix onto the orthogonal complement of the flow-invariant linear subspace. For diffusive coupling with symmetric Laplacian the linear flow-invariant manifold $\phi_1^* = \ldots = \phi_N^*$ is also the null-space of the Laplacian. In this case, the eigenvectors of the Laplacian that correspond to nonzero eigenvalues can be used to construct the projection matrix $\mathbf{V}$. For example, in the case of N characters with symmetrical all-to-all coupling with $\mathbf{L}^d = N\mathbf{I} - \mathbf{1}\mathbf{1}^T \geq 0$ we obtain $\frac{1}{2}\mathbf{V}(\mathbf{L}^d\mathbf{D}_g + \mathbf{D}_g(\mathbf{L}^d)^T)\mathbf{V}^T = N\mathbf{V}\mathbf{D}_g\mathbf{V}^T > 0$ for $\mathbf{D}_g > 0$. In this case the contraction rate is given by $\rho_{\min} = m_d\min_{\phi}(g(\phi))\lambda_{\mathbf{L}^d}^+$, with $\lambda_{\mathbf{L}^d}^+ = N$.

For general symmetric couplings with positive links with equal coupling strength $m_d > 0$ a sufficient contraction condition is: $\lambda_{\min}^+(\mathbf{L}^d)/\lambda_{\max}^+(\mathbf{L}^d) > \max_{\phi}(|g(\phi) - \text{mean}(g(\phi))|)/\text{mean}(g(\phi))$, with $\text{mean}(g(\phi)) = 1/T\int_0^T g(\phi)d\phi$. This condition was derived from the fact that for symmetric (positive) matrices M_1 and M_2 for $(M_1 - M_2) > 0$ it is sufficient to satisfy $\lambda_{\min}(M_1) > \lambda_{\max}(M_2)$. This

[1] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video0.avi

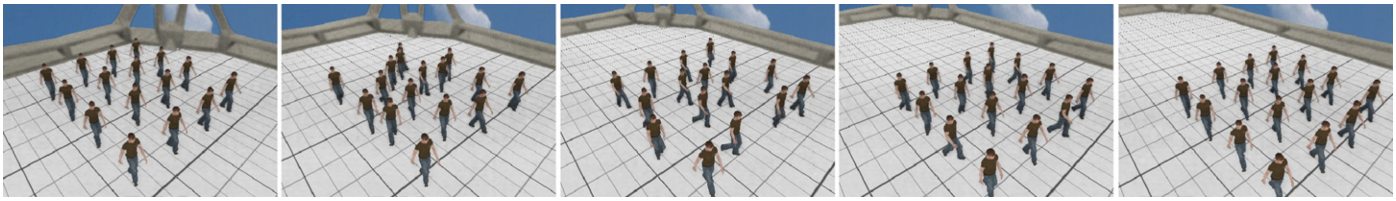

Fig. 4. Self-organized reordering of a crowd with 16 characters. Control dynamics affects simultaneously direction, distance and gait phase. See [**Demo**[7]].

sufficient condition permits to constrain the admissible coupling topologies dependent on the $g(\phi)$. Alternatively, it is possible to introduce low-pass filtering in the control dynamics to increase the smoothness of the function $g(\phi)$, see Ref. [3].

These stability bounds are illustrated by [**Demo**[2]] that shows convergent behavior of the characters when the contraction condition $m_d > 0$, $(\mathbf{L}^d)_s \geq 0$ is satisfied for all-to-all coupling. [**Demo**[3]] shows the divergent behavior of a group when this condition is violated when $m_d < 0$. In these and the next demonstrations the actual values of interaction parameters m_d, m_z, $m_\psi > 0$ (in cases, when they additionally satisfy the sufficient contraction conditions) were obtained by matching the corresponding convergence rates to those of the real human behavior in crowds [31].

(3) Step size control combined with control of step phase: the dynamics is given by Eqs. (4) and (5) with $m_d = 0$. This dynamics defines a hierarchically coupled nonlinear system (in type of Eq. 8). While the dynamics would be difficult to analyze with classical methods, the dynamics for $\mathbf{z}(t)$ that is given by Eq. (4) is partially contracting in the case of all-to-all coupling for any bounded external input $\phi(t)$ if $m_z > 0$, $\mathbf{L}^z \geq 0$, and $\boldsymbol{\omega}(t) > 0$. These sufficient contraction conditions can be derived from the requirement of the positive-definiteness of the symmetrized Jacobian applying similar techniques as above. The Jacobian of this subsystem is $\mathbf{J}(\phi, \boldsymbol{\omega}) = -m_z \mathbf{D}_g^z(\boldsymbol{\phi}, \boldsymbol{\omega})\mathbf{L}^z$, with the diagonal matrix $(\mathbf{D}_g^z(\boldsymbol{\phi}, \boldsymbol{\omega}))_{ii} = \omega_i g(\phi_i + \phi_i^0)$ that is positive definite since $g(\phi) > 0$ and $\boldsymbol{\omega} > 0$. This subsystem is (exponentially) contracting and its relaxation rate is determined by $\rho_z = m_z \min_{\phi}(g(\phi))\lambda^+_{\mathbf{L}^z}$ (in the case of all-to-all coupling) for any input from the dynamics of $\phi(t)$, cf. Eq. (5). The last dynamics is contracting when $(\mathbf{L}^\phi)_s \geq 0$ and its relaxation rate is $\rho_\phi = k\lambda^+_{\mathbf{L}^\phi}$, where $\lambda^+_{\mathbf{L}^\phi}$ is the smallest non-zero eigenvalue of $(\mathbf{L}^\phi)_s$. The effective relaxation time of the overall dynamics is thus determined by the minimum of the contraction rates ρ_ϕ and ρ_z (Fig. 4).

Demonstrations of this control dynamics satisfying the contraction conditions are shown in [**Demo**[4]], without control of step phase, and in [**Demo**[5]], with control of step phase.

(4) Advanced scenarios: a simulation of a system with stable dynamics including both types of speed control (via step size and step frequency) and step phase control is shown in [**Demo**[6]], and a larger crowd with 16 avatars simulated using the open-source animation engine Horde3d [32], is shown in [**Demo**[7]]. In this simulation an additional dynamics for obstacle avoidance and the control of heading direction was activated during the unsorting of the formation of avatars. Then this navigation dynamics was deactivated, and speed and position control according to the discussed principles result in the final coordinated behavior of the crowd. (See [**Demo**[8]].) Finally, [**Demo**[9]] shows the divergence of the dynamics for $m_d < 0$, violating the contraction condition for the step phase dynamics. The two simulations shown in [**Demo**[10]] and [**Demo**[11]] illustrate the convergence for a crowd with 49 avatars for two different values of the strength of the distance-to-step size coupling, the parameters of step phase coupling remaining constant. The development of stability bounds and estimates of relaxation times for more complicated scenarios is the goal of ongoing work.

(5) Control of heading direction: for the control of heading directions in presence of couplings that affect the step phases, the contraction conditions can be derived exploring the result on hierarchically coupled systems discussed in Section 5. For the analysis of the stability of the dynamics defined by Eq. (6) it is thus sufficient to analyze the contraction properties of the dynamics for the heading direction ψ, treating the additional term $\omega(t)g^\psi(\phi(t))$ as an external input to the ψ subsystem.

Assuming a constant goal direction, it was shown in Ref. [2] (Ch. 3.9) that the uncoupled dynamics for one character, given by $\dot{\psi} = -\omega(t)m_\psi \sin(\psi - \psi^{\text{goal}})$ is contracting in the intervals $]\psi^{\text{goal}} - \pi + 2\pi n, \psi^{\text{goal}} + \pi + 2\pi n[$, $n \in \mathbb{Z}$ for constant $m_\psi > 0$. (If $\phi(t)$ is a smooth strictly increasing function of t with the substitution $\psi(\phi(t)) = \psi(\phi)$ (and $\omega(t) = d\phi/dt$) the last differential equation can be rewritten then: $d\psi/d\phi = -m_\psi \sin(\psi(\phi) - \psi^{\text{goal}})$).

Another possibility is to realize direction control is to feed back the circular mean average direction of all characters as joint control parameter $\chi = \text{angle}(1/N\sum_i \exp[\psi_i\sqrt{-1}])$. In this case the dynamics is given by

$$\dot{\psi}_i = \omega_i(t)(\sin(\chi - \psi_i) + g^\psi(\phi(t))), \forall i \in [1 \ldots N], \quad (12)$$

which is suitable for the application of Theorem 2. This implies that the overall dynamics is contracting if the dynamics $\dot{\psi}_i = \omega_i(t)\sin(\chi(t) - \psi_i)$ is contracting for any $\chi(t)$. The same Theorem guarantees contraction, when the consensus variable χ is estimated by a low-pass filter (with time-constant $\alpha > 0$): $\alpha\dot{\chi} = -\chi + \text{angle}(1/N\sum_i \exp[\psi_i\sqrt{-1}])$. The simulation shown in [**Demo**[12]] illustrates the consensus scenario defined by Eq. (12), (without a synchronization of gait cycles).

Conclusions

The analysis and design of the dynamic properties of the formation of ordered patterns in crowds so far has been only rarely treated in computer animation, and treatments in control theory typically assume highly simplified agent models. To our knowledge, this paper presents the first systematic treatment of the dynamics of order formation in crowds using more complex agent models that include articulation of the characters during locomotion. Combining a set of specific approximations of the system dynamics with Contraction Theory as mathematical framework for the systematic treatment stability properties of complex nonlinear systems, we presented a number of examples for the derivation of stability

[2] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video1.avi
[3] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video2.avi
[4] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video3.avi
[5] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video4.avi
[6] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video5.avi
[7] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video6.avi
[8] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video7.avi
[9] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video8.avi
[10] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video9.avi
[11] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video10.avi
[12] http://www.uni-tuebingen.de/uni/knv/arl/avi/ct2012/video11.avi

bounds for nontrivial scenarios of coordinated crowd behavior during locomotion. We think that the shown examples demonstrate the feasibility of the applied approach and make it plausible that it can be extended for even more complex scenarios. Necessarily, this first exploratory study is highly incomplete and the spectrum of analyzed behaviors of crowds is still very limited. Future work will have to add other dynamical primitives to the model architecture, including ones suitable for the realization of other behaviors than locomotion. The integration of such additional components will necessitate the development of new approximations and applications of additional methods from nonlinear control theory in order to derive the relevant contraction bounds. This defines the research agenda for our future work.

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